A new approach to amplitude-and-phase interpretation of magnetotelluric data from the Carpathians

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Abstract. The magnetotelluric data is usually presented as apparent resistivity plots or MT sounding phase curves. For 2D geoelectric cross-sections, the shapes of amplitude and phase curves of MT soundings depend on the azimuth of the measurement array. Thus, MT sounding data for electric and magnetic polarization must be interpreted separately.

The magnetotelluric measurement data may be presented as the invariant of the orientation of the measurement array, as velocity of electromagnetic wave propagation energy. The amplitude and phase MT sounding curves for electric and magnetic polarization are transformed into plots of electromagnetic energy velocity versus skin effect depth.

The plots of changes of electromagnetic energy velocity versus depth may be approximated by straight line sections with different slope depending on the resistivity of a geoelectric medium. Numerous transformations of resistivity curves into velocity curves show that intersections of straight line segments approximating velocity curves correspond to boundaries of geoelectric complexes.

Key words: magnetotelluric sounding interpretation, Poynting vector, electromagnetic energy, invariants

Introduction

The magnetotelluric measurement data are usually presented as amplitude MT sounding curves and plots of the phase shift between the electric vector and magnetic vector versus frequency. One of the reasons for such form of presentation is that an apparent resistivity curve qualitatively reproduces changes of real resistivity in a geoelectric earth. In the case of 2D earth, the shape of magnetotelluric sounding curve depends on the azimuth of the measurement array. When one axis of the measurement array agrees with the uniformity axis of the earth, we get two different sounding curves, one corresponding to electric polarization and the other – to magnetic polarization. Different physical phenomena dominate there, i.e. induction in case of electric polarization, and galvanic effects in case of magnetic polarization (Berdičevski & Zdanov, 1981). The magnetotelluric sounding curves behave differently for either polarization (Kaufman and Keller, 1981) and must be interpreted using different algorithms.

The dependence of the shape of MT sounding curves on the azimuth of the measurement array does not always give the unequivocal reproduction of electric conductivity distribution in the earth. Moreover, available computer programs have been written for electric polarization or magnetic polarization, so one must know the azimuth of the uniformity axis to approximate the earth with a 2D model. A few invariants may be obtained from the impedance tensor

$$[Z]=\begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix}$$

If α is the angle of rotation of the axes, x,y,z, of the Cartesian co-ordinate system, then

$$I_1 = Z_{x'x'}(\alpha)Z_{y'y'}(\alpha) - Z_{x'y}(\alpha)Z_{y'x'}(\alpha) = Z_{xx}Z_{yy} - Z_{xy}Z_{yx}$$

$$I_2 = Z_{x'x'}(\alpha) + Z_{y'y'}(\alpha) = Z_{xx} + Z_{yy}$$

$$I_3 = Z_{x'y'}(\alpha) - Z_{y'x'}(\alpha) = Z_{xy} - Z_{yx}$$

where the superscript ' denotes axes of the measurement array after rotation (Berdičevski, 1968).

In linear algebra notation, the impedance tensor is a 2x2 matrix with complex elements. Taking this into account, Shark and Menville defined seven independent invariants of the impedance (Weaver et al., 2000). Using invariants in interpretation the information on the structure of the study Earth can be obtained. And so, skew defined by Swift in 1967 makes possible to evaluate whether the earth is 1D, 2D or 3D. Some invariants of the impedance given by Szark and Menvielle are related to galvanic effects in a non-homogenous medium.

In this paper a possibility of applying the velocity of electromagnetic energy propagating in an attenuating medium to quantitative interpretation of amplitude and phase magnetotelluric sounding curves is discussed. The electromagnetic energy velocity is the invariant of the rotation of the measurement array. Hence, it is not necessary to know the azimuth of the uniformity axis of 2D earth to quantitatively interpreted MT sounding curves.

Electromagnetic energy velocity

The energy conservation law for a plane electromagnetic wave is

Geological Survey of Slovak Republic, Dionýz Štúr Publishers, Bratislava 2001 ISSN 1335-96X

$$\dot{\overline{w}} + \overline{p} + \overline{Sz}, z = 0 \qquad (1)$$

where $\overline{w}$ is mean density of electromagnetic energy, $\overline{p}$ is mean density of energy loss, $\overline{Sz}$ is mean value of the z-component of the Poynting vector (' means a derivative). The averaging is made over the period of oscillations, so for a monochromatic wave there is $\dot{\overline{w}} = 0$ (a dot means time derivative).

Let us consider a signal of arbitrary shape. The time of arrival of the signal to the observation site is given by

$$t_o = \frac{\int t\overline{Sz}dt}{\int \overline{Sz}dt} \qquad (2)$$

The integration is made over time, t, from $-\infty$ to $+\infty$, and it is assumed that all integrals are convergent. The signal propagation velocity is defined as the derivative dz/dt_o. During its propagation, the signal may be strongly deformed. Making use of equation (1) it can be proved that

$$\frac{dt_o}{dz} = \frac{\int \overline{w}dt}{\int \overline{Sz}dt} - \frac{\int t\overline{p}dt}{\int \overline{Sz}dt} + t_o \frac{\int \overline{p}dt}{\int \overline{Sz}dt} \qquad (3)$$

(Wainstein 1963)

For a non-attenuating medium we can put $\overline{p} = 0$, and then we get for a quasi-monochromatic signal

$$\frac{dt_o}{dz} = \frac{\int \overline{w}dt}{\int \overline{Sz}dt} = \frac{1}{v} \qquad (4)$$

The expression (3) may be written for an attenuating medium in a form

$$\frac{dt_o}{dz} = \frac{1+\alpha(t_o - t_1)}{v} \qquad (5)$$

where

$$t_1 = \frac{\int t\overline{p}dt}{\int \overline{p}dt} \qquad \alpha = \frac{\int \overline{p}dt}{\int \overline{w}dt} \qquad (6)$$

For a number of simple electromagnetic processes there is

$\alpha(t_o - t_1) << 1$, and thus based on relation (3) we get for monochromatic processes

$$v = \frac{\overline{Sz}}{\overline{w}} \qquad (7)$$

where v is velocity of the signal propagating in an attenuating medium. In a sense it is the analytic continuation of the equation for the group velocity of signals in a dielectric medium.

Making use of the theorem on products and squares of complex amplitudes we get

$$\overline{Sz} = \frac{1}{2}\mathrm{Re}\{E_x(\omega)H_y^*(\omega) - E_y(\omega)H_x^*(\omega)\} \qquad (8)$$

$$\overline{\omega} = \frac{1}{4}\left\{\frac{d}{d\omega}(\omega\varepsilon')|E(\omega)|^2 + \frac{d}{d\omega}(\omega\mu')|H(\omega)|^2\right\} \qquad (9)$$

where $E_{x(y)}(\omega)$ and $H_{x(y)}(\omega)$ are components of the vectors of electric field and magnetic field, respectively, ε' is permittivity of the medium, $\varepsilon' = \varepsilon_o\varepsilon_r$; μ' is magnetic permeability of the medium, $\mu' = \mu_o\mu_r$; ε_o and μ_o are constants characteristic of the vacuum, ε_r is relative permittivity, μ_r is relative permeability, x, y, z are axes of the Cartesian co-ordinate system, and superscript* stands for a conjugate.

Expressing complex amplitudes of the vectors of electromagnetic field by their absolute values and phases we get

$$\upsilon = \frac{2}{\mu'\left(1+\varepsilon_r \cdot \left|\frac{Z(\omega)}{Z_o}\right|^2\right)} \cdot \left\{\frac{|Z_{xy}|\cdot\cos\varphi_{xy}}{1+|h_{xy}|^2} + \frac{|Z_{yx}|\cdot\cos\varphi_{yx}}{1+|h_{xy}|^{-2}}\right\} \qquad (10)$$

where $h_{xy} = \left|\frac{H_x}{H_y}\right|$; $Z_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi$ is impedance of the vacuum.

There is for a dielectric medium

$$Z_{xy} = Z_{yx} = Z(\omega) = \sqrt{\frac{\mu_o}{\varepsilon_o\varepsilon_r}} \qquad (11)$$

and hence we get

$$1+\varepsilon_r\left|\frac{Z(\omega)}{Z_o}\right|^2 \to 2 \qquad (12)$$

Under these assumptions we get

$$\upsilon \to \frac{c}{\sqrt{\varepsilon_r}} \qquad (13)$$

what corresponds to the phase frequency of a monochromatic wave in a dielectric medium with relative electric permeability, ε_r.

The impedance for a conducting medium may be expressed in terms of apparent resistivity using the known relation

$$Z(\omega) = (\omega\cdot\mu\cdot\rho_T)^{1/2} \qquad (14)$$

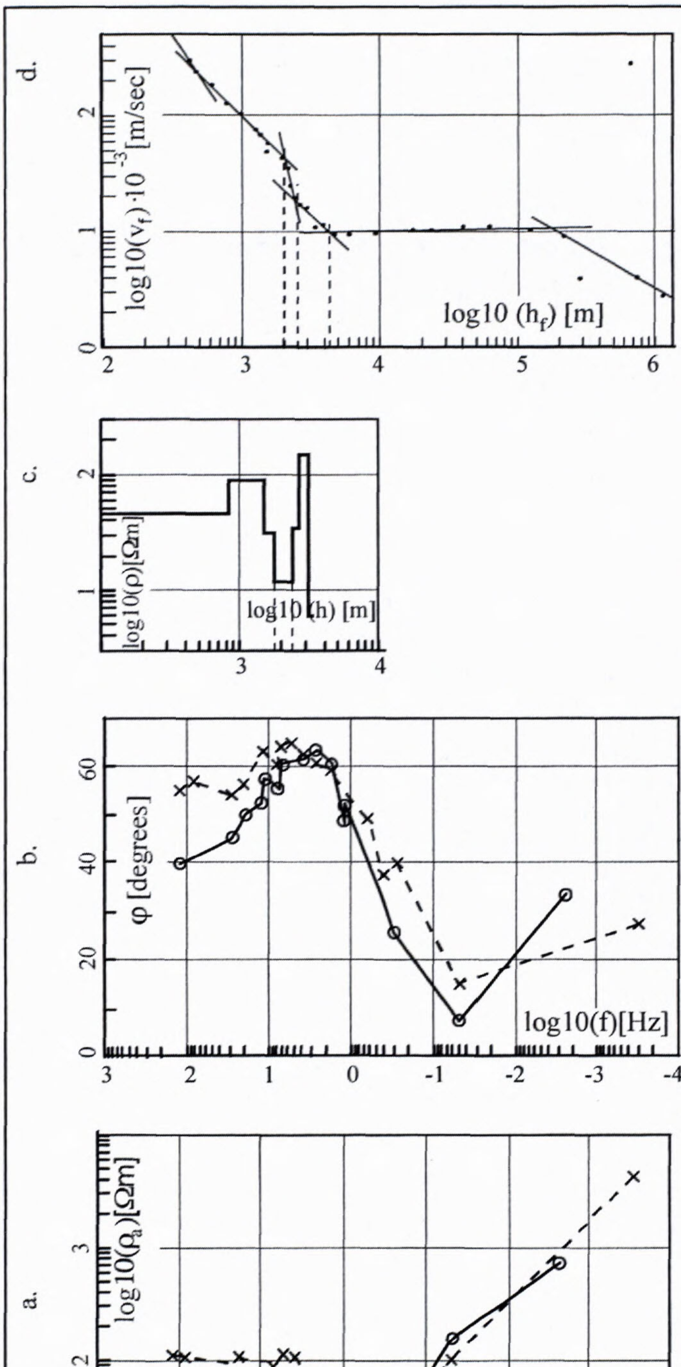
and therefore

$$\left|\frac{Z(\omega)}{Z_o}\right| \approx 0.745\cdot 10^{-5}\cdot\sqrt{\rho_T\cdot f} \qquad (15)$$

where $\omega = 2\pi f$.

It results from relation (15) that if the product $\rho_T\cdot f$ does not exceed 10^8, the ratio $\frac{Z(\omega)}{Z_o}$ is much smaller than unity and can be neglected. Then we get for the velocity of the electromagnetic energy in a conducting medium

$$\upsilon = \frac{2}{\mu}\cdot\left\{\frac{|Z_{xy}|}{1+|h_{xy}|^2}\cdot\cos\varphi_{xy} + \frac{Z_{yx}}{1+|h_{xy}|^{-2}}\cdot\cos\varphi_{yx}\right\} \qquad (16)$$



a. apparent resistivity curves, b. phase shift curves, ——— TE MODE, - - - - TM MODE, f - frequency,
c. electric log of well (Lachowice-7), ρ - resistivity, h - depth, d. apparent velocity, h - skin effect depth

According to relation (16), amplitude and phase sounding curves for electric and magnetic polarization can be transformed into one apparent velocity curve. It may be presented as a function of electromagnetic field penetration depth for an attenuating medium as

$$h_f = \left(\frac{2}{\omega \cdot \mu \cdot \sigma} \right)^{1/2} \tag{17}$$

Substituting the geometric mean of apparent resistivity (for either polarization) for conductivity in (17) we get

$$h_f = \frac{1}{2\pi} \cdot \left(10^7 \cdot \frac{\rho_m}{f} \right)^{1/2} \tag{18}$$

where $\rho_m = \sqrt{\rho_{T_E} \cdot \rho_{T_H}}$ ρ_{T_E} and ρ_{T_H} is apparent resistivity for electric and magnetic polarization, respectively.

Fig. 1 shows MT sounding curve, which was measured near the borehole in which resistivity well logging was made, and transformed into a velocity curve. It can be seen that points in which the velocity curve changes its slope correspond to depth to the top of geoelectric layers determined from resistivity logs. The transformations made for other sounding curves gave similar results.

Acknowledgements

The authors thank Director of the Department of Geology of the Ministry of Environment, as well as the Management of the Geophysical Exploration Company, Warsaw, for their consent to making use of magnetotelluric data from the Carpathians.

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