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On factors of safety and lateral curvature of a slope

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Abstract

Both natural and man-made slopes, in slope stability analysis usually considered laterally linear, belong to one of the three types: convex, linear and concave. The ratio of 2D and 3D factor of safety of a slope is related to the types mentioned. The knowledge of how significant the error — due to a computational model — might be, is of importance especially in back-calculations of landslides. Though, no quantification of such an error is available in general cases yet.

Introduction

In slope stability analysis, limit equilibrium methods are the most usual way to obtain the value of factor of safety of a slope. The problem is usually treated as two-dimensional, though recently some effort has been devoted to treat it as three-dimensional. The opinion that the minimum factor of safety in three dimensions (F_3) differs slightly from the minimum factor of safety in two dimensions (F_2) of the same slope may be well agreed with. Obviously, the more realistic three-dimensional approach will raise the accuracy of stability analysis.

Recently, a three-dimensional limit equilibrium analysis of a slope has been presented by Chen and Chameau (1983); the failure surface assumed to be known. On the basis of limit equilibrium state search for minimum factor of safety Leshchinsky, Baker and Silver (1985) use variational analysis. The way to extend to three dimensions the method described by Bishop (1955) is given by Hungr (1987). Finally, Cavounidis (1987) reasons that the F_3/F_2 ratio of a given slope is always greater than or equal to unity (Hutchinson and Sarma, 1985 express similar opinion).

All the methods mentioned assume rotational failure surface symmetrical with respect to vertical plane and the direction of movement parallel to this plane of symmetry. Only cases of simple slope geometry and homogeneous soil profile have been solved. Furthermore, in every case studied the slope has been assumed to be laterally linear i. e. two straight lines — the crest and the toe of a slope — have been defining plane surface of a slope (Fig. 1).

Three types of slopes

In reality, most natural slopes and some artificial ones, too, are of more complex geometry. With respect to lateral curvature slopes may be regarded as concave (A), linear (B) or convex (C) type (Fig. 2).

Concave type (A) in the nature is usually a sign of low stability environment. It develops in relatively soft rocks and soils with shear strength parameters, that imply lower value of factor of safety. Slopes in clay soils, rear scarps of landslides are examples of nature origin and slopes of circular foundation pits of man-made one. Záruba and Mencl (1974) point out that the three main normal stresses are approaching the limit state $\sigma_1 = \sigma_2 > \sigma_3$. During deformation distances between soil particles are decreasing and shear strength increasing.

Linear type (B) often originates due to discontinuity pre-disposition of rocks and soils and may be a subject to further development into types A, C. This is the most usual form of artificial slopes. Factor of safety value is intermediate compared with the types A, C. The state of stress corresponds to $\sigma_1 > \sigma_2 > \sigma_3$.

Convex type (C) in the nature is usually a sign of high stability environment. It develops in relatively hard rocks and soils with high shear strength parameters that imply higher value of factor of safety. Slopes in weathered coarse-grained rocks and their eluvium in the nature and slopes of some compacted fills or waste heaps of artificial origin are examples. Záruba and Mencl (1974) suppose that the three main normal stresses are approaching the limit state $\sigma_1 > \sigma_2 = \sigma_3$.

During deformation distances between soil par-

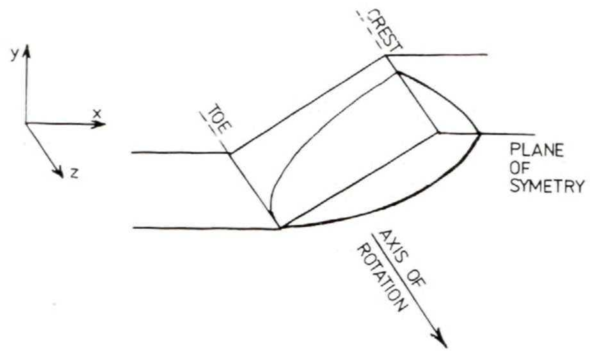

Fig. 1. Three-dimensional problem (usual assumptions).

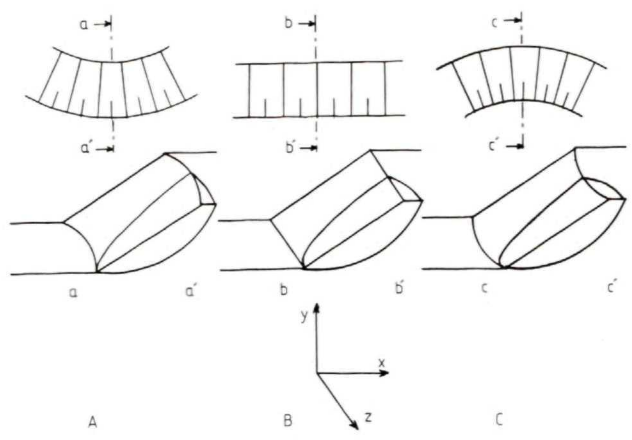

Fig. 2. Slope morphology. Laterally concave (A), linear (B), and convex (C) type.

ticles are increasing and shear strength decreasing.

In the following text, minimum factors of safety F will be indexed A, B and C in accordance with the slope types mentioned. 2D and 3D cases will be indexed 2 and 3, respectively.

Comparison of the factors of safety

To compare factors of safety between the A, B and C type of a slope it will be assumed that:

(a) Geological conditions and soil properties are the same in all three cases. Geometry of the slopes and the failure masses is in the three cases the same except the lateral curvature.

(b) For a given case there exists only one vertical cross-section which yields the 2D minimum factor of safety (in symmetrical cases this section corresponds to the plane of symmetry). Then the cross-sections a—a', b—b' and c—c' in cases A, B and C, respectively, are identical (Fig. 2), so are the 2D failure surfaces and so are the 2D minimum factors of safety. It can be written that

$$F_{2A} = F_{2B} = F_{2C} \qquad (1)$$

(c) Now, to describe relationship between the F_2 and F_3 of the A, B and C slope types it will be assumed that the direction of movement of any particle of the failure mass is always parallel to the X—Y plane (Fig. 2), that is, to the "worst" cross-section (validity of this assumption will be discussed). This enables the proof given by Cavounidis (1987) that

$$F_3/F_2 \geqslant 1 \qquad (2)$$

to be accepted and extended to A and C slope types. In brief, the proof offered by Cavounidis (1987) is based on consideration that

$$F_3 = \frac{\int_z R\,dz + E}{\int_z D\,dz} \qquad (3)$$

where R and D are resisting and driving moments per unit length, respectively, and E is additional resistance due to side forces and end resistance. Now, if the "worst" cross-section yields

$$F_2 = \frac{R_0}{D_0} \qquad (4)$$

in any other cross-section the ratio R_i/D_i must be greater than or equal to F_2, and after integration therefore

$$F_3 \geqslant F_2 \qquad (5)$$

which is equivalent of formula (2).

From this, for the slope types results

$$F_{3A} \geqslant F_{2A} \qquad (6a)$$
$$F_{3B} \geqslant F_{2B} \qquad (6b)$$
$$F_{3C} \geqslant F_{2C} \qquad (6c)$$

Any rigorous limit equilibrium analysis must respect both moment and force conditions of equilibrium. But, as Spencer (1973) put it, the approach adopted in obtaining the equilibrium equations does not affect the final solution. Ambrož (1987) confirmed this statement deriving another equilibrium equations to obtain identical solutions.

Overall moment equation (3) may be thus supplemented with a force equation written in a similar generalized form (7)

$$F_3 = \frac{\int_z P\,dz + E'}{\int_z S\,dz} \qquad (7)$$

where P and S are passive and shifting forces per unit length, respectively, E' having similar meaning as E in equation (3).

The extent of both passive and shifting forces may be — using generalized form — expressed as

$$P = \int_{x_1}^{x_2} (\sigma' \tan\varphi' + c')\, f(x)\, dx \qquad (8)$$

$$S = \int_{x_1}^{x_2} \tau\, g(x)\, dx \quad (9)$$

σ' being effective normal stress, τ shear stress in a direction of movement, φ and c' effective shear strength parameters, f(x) and g(x) functions combining requirements of equilibrium and a formula to obtain arch length, x_1 and x_2 shear surface boundaries. The values of resisting and driving moments are expressable in a very similar way.

Inserting equations (8), (9) into (7) resp. (3) and observing the influence of geometrical differences between the slope types A, B and C on values of τ resp. (σ' tan φ' + c'), it becomes apparent that

$$F_{3A} \geqslant F_{3B} \geqslant F_{3C} \quad (10)$$

because τ, mostly influenced by weight of columns, increases in case C while (σ' tan φ' + c'), influenced mainly by area of failure surface, increases in case A.

The slope types and F_3/F_2 ratio

From formulas (1) and (10) results that

$$F_{3A}/'_{2A} \geqslant F_{3B}/F_{2B} \geqslant F_{3C}/F_{2C} \quad (11)$$

or, expressed by means of differences

$$\Delta F_A \geqslant \Delta F_B \geqslant \Delta F_C \quad (12)$$

$$\text{where } \Delta F = F_3 - F_2 \quad (13)$$

That is to say, that the more concave (or the less convex) the slope, the greater the value of F_3/F_2 ratio (or the difference Δ F).

In agreement with Chen and Chameau (1983), the three-dimensional effects are more significant at smaller length of the failure mass.

Therefore the most significant errors arisen from 2D stability analysis concern slopes of laterally concave type (the length of the failure is also smaller here than at linear or convex slopes in most cases).

Conclusions

A few details resulting are useful to be kept in mind when carrying out backcalculations of landslides:

1. In most cases, two-dimensional back-analysis of a landslide is accomplished and the shear strength parameters φ_2, c_2 obtained are slightly greater than the real parameters φ, c.

$$\varphi_2 \text{ resp. } c_2 \geqslant \varphi \text{ resp. } c \quad (14)$$

Calculations now resume evaluating the efficiency of remedial measures. If a 2D method is used, the error in φ_2, c_2 values may be considered eliminated, and the final calculated factor of safety F_2 corresponding to the real factor of safety.

$$F_2 \doteq F_{real} \quad (15)$$

If a 3D method would be used, the error in φ_2, c_2 values would not eliminate and the final factor of safety F_3 would appear greater than the real factor, that is on the unsafe side.

$$F_3 \geqslant F_{real} \quad (16)$$

2. To use a three-dimensional back-analysis may appear neck-breaking and most likely will not be accurate with respect to complex natural conditions. Shear strength parameters obtainable φ_3, c_3 are equal to the real ones

$$\varphi_3 \text{ resp. } c_3 = \varphi \text{ resp. } c \quad (17)$$

They are used for further 3D method computation the resulting factor of safety F_3 is equal to the real one

$$F_3 = F_{real} \quad (18)$$

If a 2D analysis using φ_3, c_3 values would be executed, of course the final factor of safety would be slightly smaller than the real

$$F_2 \leqslant F_{real} \quad (19)$$

that is, on the safe side.

3. It still remains left to discuss the assumption (c), that the direction of movement of all sliding mass particles is parallel to one plane. As it has been said above (description of A and C slope types), during deformation the distance between particles changes and so does the shear strength and the factor of safety. To touch this theme at least with a few words: the above review concerns static factors of safety, every comparison based on equality of soil properties (mainly shear strength). Thus, comparative forms (1), (6), (10), (11) and (12) are valid.

Comparing of factors of safety in various stages of deformation (when the shear strength might differ) may result in appearance of paradox observations like (20)

$$F_{3C} < F_{2C} \quad (20)$$

which means that the reality is misinterpreted (φ_2, c_2 parameters are differing from φ_3, c_3 in fact). As in this case

$$F_{2C} \neq F_{2B} \quad (21)$$

inequality (20) would be

$$F_{3C} > F_{2C} \quad (22)$$

but

$$F_{3C} < F_{2B} \quad (23)$$

Similarly for the A-type of a slope it may result

$$F_{3A} >> F_{2A} \quad (24)$$

which is incorrect but not so striking.

All the previous text shows the fundamental importance to define the way a factor of safety has been computed every time a value of this factor is given.

References

Ambrož, J. 1987: Some possibilities of rigorous computation of factor of safety of a slope (in Czech). *Proc. 5th Conf. Small Computers in Geomechanics, Prague, 59—70.*

Bishop, A. W. 1955: The use of the slip circle in the stability analysis of slopes. *Géotechnique, 5, 1, 7—17.*

Cavounidis, S. 1987: On the ratio of factors of safety in slope stability analyses. *Géotechnique, 37, 2, 207—210.*

Chen, R. H., Chameau, J. L. 1983: Three dimensional limit equilibrium analysis of slopes. *Géotechnique, 33, 1, 31—40.*

Hungr, O. 1987: An extension of Bishop's simplified method of slope stability analysis to three dimensions. *Géotechnique, 37, 1, 113—117.*

Hutchinson, J. N., Sarma, S. K. 1985: Discussion on Three-dimensional limit equilibrium analysis of slopes. *Géotechnique, 35, 2, 215.*

Leshchinsky, D., Baker, R., Silver, M. L. 1985: Three dimensional analysis of slope stability. *Int. J. Numerical and Analytical Methods in Geomechanics, 9, 199—223.*

Spencer, E. 1973: Thrust line criterion in embankment stability analysis. *Géotechnique, 23, 1, 85—100.*

Záruba, Q., Mencl, V. 1974: Engineering geology (in Czech). *Prague, Academia, 80—82.*

Stupně bezpečnosti a laterální zakřivení svahu

Při výpočtech stability svahu metodami mezní rovnováhy se nejčastěji úloha řeší jako rovinná, dvojrozměrná (2D). Takto vypočtený stupeň bezpečnosti F_2 se poněkud liší od stupně F_3, odvozeného z trojrozměrného (3D) modelu. Autoři výpočtů 3D většinou uvažují (kromě dalších zjednodušení) laterálně lineární průběh svahu (obr. 1). Skutečné svahy mohou být označeny podle svého laterálního průběhu jako konkávní typ A (zeminy s malou schopností vytvářet stabilní svahy, např. odlučné oblasti sesuvů v jílech), lineární typ B (většina umělých svahů) či konvexní typ C (materiály schopné vytvářet stabilní svahy, např. eluvia hrubozrnných hornin). Minimální stupně bezpečnosti jsou dále opatřeny indexy 2 nebo 3 (úloha 2D nebo 3D) a A, B, C podle uvedených typů.

Pro srovnání uvedených případů musí být geologické podmínky a geometrie svahu (s výjimkou zakřivení) shodné (obr. 2). Odtud plyne (1). Pohybuje-li se každá částice paralelně s rovinou X-Y, lze vztah (2) odvozený Cavounidisem (1987) rozšířit na typy svahu A, C (6a), (6b), (6c). Momentovou rovnici (3) lze při staticky přesném řešení rozšířit o silovou rovnici (7). Pasívní momenty R a aktivní D jsou vystřídány pasívními silami P a aktivními S (8, 9), kde f (x) a g (x) jsou funkce vyhovující požadavkům mezní rovnováhy na oblouku smykové plochy. Po dosazení (8), (9) do (7), příp. (3), je zřejmé, že smykové napětí τ v důsledku tíhy sloupců roste k případu C, zatímco smyková pevnost (σ' tg φ' + c') v důsledku plochy základny sloupců roste k případu A, a odtud plyne (10). Stejně tak přídavné pasívní momenty E resp. síly E', způsobené bočními silami v tělese, rostou k případu A.

Z (1) a (10) vyplývají vztahy (11), (12), (13) — čím více konkávní (méně konvexní) svah, tím větší poměr F_3/F_2. Efekt se zvyšuje, zkracuje-li se délka svahu.

Nejvýznamnější chyby výpočtů 2D tedy vznikají u krátkých konkávních svahů.

Při zpětných výpočtech sesuvů je tedy vhodné mít na zřeteli, že:

1. Pevnostní parametry φ_2, c_2 zjištěné zpětným výpočtem 2D jsou vyšší než skutečné (14). Při dalším výpočtu způsobem 2D, který uvažuje stabilizující prvky, je chyba přibližně eliminována, takže platí (15). Výpočtem metodou 3D by došlo k (16), což je na nebezpečné straně.

2. Provést zpětný výpočet způsobem 3D je obtížné. Zjištěné pevnostní parametry φ_3, c_3 by odpovídaly skutečným (17). Při dalším výpočtu způsobem 3D, uvažujícím stabilizační prvky, je stupeň bezpečnosti reálný (18). Při užití způsobu 2D je výpočet podle (19) na straně bezpečnosti.

Protože během svahového pohybu se mění vzdálenosti částic, a tím smyková pevnost, může při porovnávání různých stádií deformace zdánlivě platit (20). Správná interpretace skutečnosti v tomto případě by byla podle (21), (22), (23).

Porovnávací vztahy (1), (6), (10), (11), (12) platí totiž pro statický stupeň bezpečnosti, který vychází ze shodných vlastností zemin (zvláště smykové pevnosti) v posuzovaných případech.

Předložený text ukazuje zásadní důležitost toho, aby byl — kdykoliv se udává hodnota stupně bezpečnosti— také uveden způsob, jakým byl stupeň bezpečnosti vypočítán.